Mr. Michael T. Davis Calculus This quiz is due at the start of class on Tuesday, February 21	Section 4.2 Take-Home Quiz February 17, 2017

Name: Solutions

1. Find y' given $x = \sin y$

$$1 = \cos y \cdot \frac{dy}{dx}$$

$$\frac{1}{\cos y} = \frac{dy}{dx}$$

$$\sec y = \frac{dy}{dx}$$

2. Find $\frac{dy}{dx}$ given $2x^2 + xy = y$

$$4x + y + x\frac{dy}{dx} = \frac{dy}{dx}$$

$$x\frac{dy}{dx} - \frac{dy}{dx} = -4x - y$$

$$\frac{dy}{dx}(x-1) = -4x - y$$

$$\frac{dy}{dx} = \frac{-4x-y}{x-1}$$

3. Find y' given $\sin y = y^2 - x$

$$\cos y \cdot y' = 2y \cdot y' - 1$$

$$y' \cos y - 2y \cdot y' = -1$$

$$y'(\cos y - 2y) = -1$$

$$y' = \frac{-1}{\cos y - 2y}$$

4. Find y' and the slope of the curve of $x^3 + y^3 = 9$ at the point $(-1,2)$

$$3x^2 + 3y^2 y' = 0$$

$$3y^2 y' = -3x^2$$

$$y' = \frac{-x^2}{y^2}$$

at $(-1,2)$ $y' = \frac{-(-1)^2}{2^2} = \frac{-1}{4}$

5. Find the equations of the lines that are (a) tangent and (b) normal to the curve of $y^2 - 2x - 4y = 1$ at the point $(-2,1)$

$$2y \cdot y' - 2 - 4y' = 0$$
$$2y \cdot y' - 4y' = 2$$
$$y'(2y-4) = 2$$
$$y' = \frac{2}{2y-4}$$

at $(-2,1)$ $y' = \frac{2}{2-4} = -1$

(a) $y - 1 = -(x+2)$

(b) $y - 1 = (x+2)$

6. Find the equations of the lines that are (a) tangent and (b) normal to the curve of $2xy + \pi \sin y = 2\pi$ at the point $\left(1, \frac{\pi}{2}\right)$

$$2y + 2xy' + \pi \cos y \cdot y' = 0$$
$$2xy' + \pi \cos y \cdot y' = -2y$$
$$y'(2x + \pi \cos y) = -2y$$
$$y' = \frac{-2y}{2x + \pi \cos y}$$

at $\left(1, \frac{\pi}{2}\right)$ $y' = \frac{-2\left(\frac{\pi}{2}\right)}{2(1) + \pi \cos\left(\frac{\pi}{2}\right)}$

$= \frac{-\pi}{2}$

(a) $y - \frac{\pi}{2} = \frac{-\pi}{2}(x-1)$

(b) $y - \frac{\pi}{2} = \frac{2}{\pi}(x-1)$

7. You must show all the steps in your work. Given the relation $x^2 - xy + y^2 = 1$, then $\frac{dy}{dx} =$

a. $\frac{2x+y}{x-2y}$ b. $\frac{y+2x}{2y-x}$ c. $\frac{2x}{x-2y}$

d. $\frac{y-2x}{2y-x}$ e. $\frac{y+2x}{x}$

$$2x - y - xy' + 2y \cdot y' = 0$$
$$2y \cdot y' - xy' = y - 2x$$
$$y'(2y - x) = y - 2x$$
$$y' = \frac{y-2x}{2y-x}$$

8. You must show all the steps in your work. Given the relation $x^2+y^2=64$, then $\frac{d^2y}{dx^2}=$

a. $\frac{y^2-x^2}{y^3}$ b. $-\frac{y^2+x^2}{y^3}$ c. $\frac{y^2+x^2}{y^3}$

d. $\frac{-y^2+x^2}{y^3}$ e. $\frac{-y^2-x^2}{y^2}$ f.

$$2x+2y\cdot y'=0$$
$$x+y\cdot y'=0$$
$$y'=\frac{-x}{y}$$
$$y''=\frac{y(-1)-(-x)y'}{y^2}$$

$$y''=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}$$
$$y''=\frac{\frac{-y^2}{y}-\frac{x^2}{y}}{y^2}$$
$$y''=\frac{-y^2-x^2}{y^3}=-\frac{y^2+x^2}{y^3}$$

9. Find $\frac{d^2y}{dx^2}$ given $y=\sin^2(3x-5)$

$$\frac{dy}{dx}=2\sin(3x-5)\cos(3x-5)\cdot 3=6\sin(3x-5)\cos(3x-5)$$
$$\frac{d^2y}{dx^2}=6\sin(3x-5)(-\sin(3x-5)\cdot 3)+6\cos(3x-5)\cdot 3\cdot\cos(3x-5)$$
$$=-18\sin^2(3x-5)+18\cos^2(3x-5)$$

<u>Optional Extra Credit (you may continue your work on the back of this paper)</u>

You must show all the steps in your work. Given the relation $x^2-xy+y^2=1$, then $\frac{d^2y}{dx^2}=$

From #7 $y'=\frac{y-2x}{2y-x}$

$$y''=\frac{(2y-x)(y'-2)-(y-2x)(2y'-1)}{(2y-x)^2}$$

$$y''=\frac{(2y-x)\left(\frac{y-2x}{2y-x}-2\right)-(y-2x)\left(\frac{2(y-2x)}{2y-x}-1\right)}{(2y-x)^2}$$